Some Considerations Regarding the Phenomenological Relationship Between Music And Mathematics

Gerardo del Cerro Santamaría, *Social Science at The Cooper Union for the Advancement of Science and Art in Manhattan, New York, Email: gdelcerro@gmail.com*

doi: 10.22545/2015/00063

This papers explores the phenomenological relationship between music and mathematics. The paper starts by giving some examples that may clearly validate the contribution of maths intuitive dimension to some of the most important historical discoveries of this discipline. We will try to show how these intuitive visions present an order that we might associate with the idea of beauty: we are talking about an aesthetic way of thinking. Afterwards we will suggest that this reasoning- commonly attached to music and mathematics- is not a mere abstract construct, but it has practical consequences: beauty contributes to truth and truth contributes to beauty. Finally, we will argue that what essentially unites both disciplines is the process of creative research and the constructions of metaphors, a process that permits the translation of symbolic codes.

Keywords*:* music, mathematics, intuitive reasoning, phemomenology, truth, beauty, metaphors.

1 Introduction

One would not consider the argument regarding the formal relationship between music and mathematics to be extremely complicated nor excessively original. Since Pythagoras we have evidence of the analytical relationship between both fields of knowledge. Physical science has mathematically coded the sonic experience. There are studies that try to explain a few musical compositions through mathematical operations. It is to some extent evident that these two fields are somehow related and that they organize their elements in a hierarchy: the numerical series, for example, are organized in a similar way to sound series..

Nevertheless, the descriptive or analytical procedures cannot completely explain the true essence of the musical experience. In this point you can start to think that mathematical enterprises are now totally the opposite to that of mathematics, and that theres nothing more distinctive than music (an unexplicable experience) and mathematics (a deductive building). Is this true? We know that music can be formalized – can math be experienced in an intuitive way? And if this is so, which ideas can be useful in order to trace the connections between these two disciplines?

In the pages that follow I would like to develop some conjectures as means of trying to give an answer to these questions. I will start by giving some examples that may clearly validate the contribution of maths intuitive dimension to some of the most important historical discoveries of this discipline. I will try to show how these intuitive visions present an order that we might associate with the idea of beauty: we are talking about an aesthetic way of thinking. Afterwards I will suggest that this reasoning- commonly attached to music and mathematics- is not a mere abstract construct, but it has practical consequences: beauty contributes to the truth and truth contributes to beauty. Finally, I will argue that what essentially unites both disciplines is the process of creative research and the constructions of metaphors, a process that permits the translation of symbolic codes.

1. Let's start with a historical example. Henri Poincaré, the French mathematical celebrity, described in a conference about mathematical creations the existence of certain functions referred to as "fuchsians".

"I left Caen, where I lived, for a geological expedition sponsored by the Mines School. The episodes during the travel made me forget my mathematical work. When we passed Coutances, we went deep into some unknown fields until we finally made a stop. In the exact moment that I put my feet on the ground, with no apparent connection to what I was thinking during my trip, the thought that the mathematical transformations that I have used to define the fuchsians functions were identical to that of non euclidean geometry. I didnt verify the idea (nor had the time to, I started an ordinary conversation with a fellow traveler), but I had the perfect certainty about its validation. As soon as I got back to Caen, I verified the results during my free time".

This story of "illumination" or revelation is more interesting because it's not atypical. Carl Friedrich Gauss tells a similar story of how he found the demonstration of a theorem which proof he has been looking for years:

"Finally, after two days, I found it, and it wasnt because of my daring efforts, it was merely God's grace. As if a sudden light shook me, and suddenly the problem was solved. I am not capable of understanding which was the thread that organized my previous thoughts and made my success possible."

Jacques Hadamard, in an influential book titled An Essay on The Psychology of Invention in the Mathematical Field, compares the ability of some mathematicians to instantaneously seek the solution to a problem with a Mozart's letter where he describes how he is capable of seeing his symphonies completed in his head before translating them to paper. The act of "seeing" is not literal, nor pictoric. It is an aptitude of comprehending in its totality.

Poincaré explained that the capacity of "seeing" implicated a series of rules that are "extremely delicate and subtle. It is nearly impossible to define them; you can only feel them, you can't formulate them. Poincar considered the capacity of mathematical creation as a certain "emotional sensibility".

"It may result surprising to observe how the emotional sensibility is invoked in mathematical demonstrations, but it is only appealing to the intellect. But to think of it that way would imply to neglect the sense of beauty in mathematics, the sense of harmony within the forms and the numbers, the sence of geometrical elegance. We are talking about a true authentic aesthetic sentiment that belongs to the realm of emotional sensibility."

2. We may add that this "vision" that provides completion and certainty has something to do with our ideas of beauty. And while in music the idea of beauty occupies a prominent place; perhaps we should learn a bit more about its place in mathematics. Lets suggest, for a start, that a mathematical test, if it's successful, has some kind of "rhythm". This rhythm is evident in the ways that the ideas are presented, in the types of punctuation that are being used, in the imitations of generic models. We can say that there are moments of drama, of suspense and surprise, that there are different styles of compositions in mathematical tests. Let's see what Ludwig Boltzmann has to say about his great colleague J.C. Maxwell:

"The same way a musician is able to recognize within a few seconds the music of Mozart, Beethoven or Schubert, a mathematician is able to recognize Cauchy, Gauss, Jacobi, Helmholtz, or Kirchhoff by just reading the first few pages. French writers exhibit great formal elegance, whereas the English writers, especially Maxwell, exhibit their great dramatic sense. Who doesn't recognize Maxwell's descriptions of the dynamic theory of gases?

The variations within velocity majestically appear; then later, it makes way for the state equations and then on another side, the equations of a central field movement. Thats how chaos is born. Suddenly, we listen to some distant drums, four hits: "add n=5". The diabolical spirit V (the relative velocity of both molecules) disappears; and like in music, a dominant figure in the bass is silenced, and what seemed unsurmountable is surmounted, like a work of magic... It is not time to question ourselves about this or that substitution. If you have not been swept by the development of the events, leave the paper aside. Maxwell doesnt write programmed music with explicative notes. A result is followed by another one very rapidly until finally, as an unexpected climax, we reach to the conditions of the thermical equilibrium combined with the expressions of the transpositions of coefficients. Suddenly the curtain falls in.

One does not have to understand the details of Maxwells argument to appreacite the spirit of the explanation, it appears as if a romantic spirit were behind the descriptions of those equations, and it may not be chance the imagery of Romantic music that Boltzmann used to describe it with. In effect, it is possible to perceive the rhythm even in the evolution of the simplest mathematical operations, once we are familiar with them. This style is a dimension of the emotional sensibitlity of mathematics and beauty, one of their goals. Bertrand Russell who, together with A.N. Whitehead, dedicated his energies to the systematization of the arythmetics through the usage of symbols of mathematical logic, stated that: "Mathematics, correctly conceived, possess not only truth value, but also supreme beauty –a cold and austere beauty, like that of a sculptur...The authentic spirit of pleasure, of exaltation, the power of being more human, which is the primary characteristic of supreme excellence, can be found in mathematics as much as in music."

What has been said about the literary style of some of the greatest mathematicians in history can also be said about the particular tutoring style in which great masters transmit the teachings of the great creators. If we are talking about "style", this is clearly revealed in their pedagogic ways, that is to say, in the speech presentation of these arguments and in the expected elegant way of showing their solutions. Here – and thousands of students know this – the teachings in mathematics can fail rapidly due to "aesthetic" reasons: lack of performance. It is not exaggerated to say that the rhythm of a presentation about mathematical topics affects their comprehension completely. And it is not rare to compare the experience and expectations of a class audience in Calculus class with that of, for example, a visit to a museum. In both cases we are facing objects of knowledge (the work of art, the mathematical demonstration) whose comprehension goes beyond the mere description or anaylisis: the mistery, the beauty, the rhythm, the vision are resulting primary components that will help us understand in an intimate and lasting way what we have before our hands.

Lets think of any logic game. The intrinsic sense of a game does not rest too much on the description of their rules (the ones that are internalized) but in the comprehension of the proportions, the recognition of behavioral patterns and the typical results associated to those models. The "texture" of a logic game is a net of cognitive expectations through which we build a "vision", seeming and effective, of its development.

The beauty of the game consists in the simplicity of the reasonings that we are capable of developing, and in our capacity to avoid embroiling ourselves in detail calculations. This is a typical way of thinking in mathematics. The emotional sensibility and the beauty embroiled in the great sense of freedom and elegance given to us by the fact of guessing a method of rapid solution while facing the enormity of the task that comes rushing over us. As Lord Rayleigh said in the 19th Century: "Some demonstrations provoke assent. Others appeal to the intellect; they evoke pleasure and inspire an unstoppable desire of saying "Amen, Amen".

3. I would add that this aesthetic line of thinking has strong repercussions in the way of knowing. The elegance and beauty as creating behaviors imply a clarification of something essencial in the object of knowledge, and this also implies a change in our conception of the object. This point is rather important; it means that we are not only talking about the intuitive comprehension of the mathematical beauty, but also its observation in real life. In other words, we speak of the beauty of what is true, and we can speak this way because we assume that that specific beauty is revealing something generic, attributed to a type of objects, not only to a specific or one in particular. The most eloquent example of this mathematical debate between reality and creation, between intervention and interpretation, is the *pi number*.

Pi expresses the reason between a circumference and its diameter: a constant reason independently from the size of the circumference and the length of its diameter. Besides, we're speaking about a number that *can never be calculated with absolute* *exactitude*. *Pi* is more than an irrational number: its a transcendental number. This inaccuracy in which we are permitted to approximate to Pis calculus has not prevented the creation of mathematical formulas of this elegant number throughout history. Leibniz found a relationship between pi and a numerical series; and Euler was able to express it in relation to the imaginary number *e*. There is an establishing harmony in both cases by which all of the elements are connected with each other; this is the origin of mathematical beauty.

Stated differently, the mathematical argument is beautifully constructed when you can say it has a purpose, a design, clearly different from the processes of reasoning that originated it. This way, and only this way, we have the intimate impression that the nature of things is constructed for our own contemplation and enjoyment. The discovery's objective and the scientific creativity, particularly that of mathematics, is an aesthetic objective: to express the harmonies existing in nature which, as Barrow said, are far from being evident and directly observables. In this sense we can confirm that to the extent that science lacks artistic sense is an incomplete science. Or, with Kant, we can say that science develops by analogy with art.

We have proposed the idea that truth contributes to beauty – the idea that what's true is beautiful since it reveals non-evident connections among real elements. There are those who have suggested an inverse relationship – the idea that beauty contributes to the truth of things and that it takes precedence over them. Thereby, Paul Dirac stated with clarity: "It's more important to find beauty in the equations themselves than to be able to explain an experiment. The experiments, after all, may be wrong". Theres not a doubt that this statement may result controversial, especially because it indicates a subordination of ethics to aesthetics. We can also start to think with ease about examples of beautiful things – beautiful theories – that aren't true: without going any further, the Ptolemaic model of the universe.

Even though it looks extremely difficult to be established, as shown in the comments made above, there is an essential connection between art, nature and mathematical thinking – we are referring to the quality of expressing the general, which is shared by aesthetic judgement and mathematical reasoning. Naturally, we are not stating that aesthetic judgement owns a universal character (we would have to prove this); but we are not proposing either to treat any notion of beauty as a statement related to taste, in the postmodern sense. We simply say that the judgement towards beauty can be treated as if it were universal and sensed as objective.

Unlike social constructionists, for whom there is no essence outside of speech, nor speech not socially constructed, we think that aesthetic attitudes of thinking as the ones given in mathematics exhibit the virtue of risking an attempt to define relations and essential structures that are objectively existing. The feeling of beauty in mathematicas has to do with the revelation of the universal, the discovery of the similar, the construction of order, the establishment of reasoning and proportions, the articulation of a whole and its parts, and the parts inside of a whole. The idea of beauty in music obeys similar patterns.

4. It is possible to dig deeper in this phenomenological connection that we are trying to trace. Firstly, the idea of proportion (the establishing of relations between different eareas) is closely linked to the idea of analogy – this is not but a special case of that analogy that implies a consonance between the parts and the whole. This idea of analogy (or geometric proportion) as aesthetic construction is the solid base of harmony, common to the ways of thinking and operating in art and science. Secondly, the ideas of analogy and metaphors are intimately linked, for a metaphor is an unexpected and condensed analogy. Aristotles granted great importance to the metaphor:

"The greatest thing of all is to be a master of the metaphor. It's the only thing that one can learn from others; and it is a sign of genius and originality, because a good metaphor implies intuitive perception of similarities in dissimilar things."

The search of metaphors is what links music and mathematics. The constant search that both activities consist of, give a wide possibility of feeling an aesthetic joy, of experimenting beauty, simply because we use our imagination. In mathematics we treat nature as an analogy of art; in music we treat art as an analogy of nature. But what's common in both areas is that we can symbolically translate between two different worlds of knowledge. In the words of Kant, "the imagination is a powerful entity of creation of a second nature species from material provided by real nature."

But this process of symbolic translation, of metaphorical creation, it's produced by specific ways that are very differently in each case. Human beings experience the sonic material – and not mathematics – as a temporal experience. It's precisely this "being in time" – as Heidegger would say – which gives music a sensual presence that is lacking in mathematics. There may be beauty in sound itself even though its not accompanied by a musical order because the beauty of sound is the same as that of a form – it has an internal harmony, it provokes presence and evokes absence; implies and complies. From a technical point of view, every sound is composed of harmonics that are mobilized and they accompany the original sound in its manifestation; even more, every sound can be organized as components of one or various close tonalities. From an orchestral point of view, every orchestra has or acquires a sound of its own as a musical (not strictly technical) project. From a socio-historical perspective, every sound involves a choice: the deployment of individual and collective identities, of the historical memory, future projects, or their absence.

Therefore, the sense of beauty in music, its sensual presence, is but the metaphorical construction (on the part of listener, composer, and interpreter) of networks of sonic relationships evoked by the sound currently being produced. A composition is a construction of models and proportions, of a determined musical form that will keep established conventions but will try to transcend hem meaningfully. The ear, like the mind that tries to understand mathematical arguments, operates in asimilar way, through means of relative constructions from the sounds you hear in reality. The ear does not exactly follow the sound flow in time: it anticipates and it lingers; progresses and returns; it sways. It establishes specific and original connections as much as in the structure of individual feeling of every listener as with the external contexts in which the individual experience develops. Occasionally, the connections can be so dense and vivid that the auditory attention gets lost and the process of symbolic translation acquires a life of its own – that is when the transformation of art in nature (of which we spoke earlier) occurs.

These phenomenological processes are virtually impossible to codify. And so the score is merely a two-dimensional and static representation unable to reflect every aspect of creation that we are sketching – the space of creation of the listener, and the listener that composes and the one that interprets. That's how we construct metaphor in music. Thats how we can enjoy a musical piece, anticipating the memory of fragments and previous ideas to fragments and further ideas in which we can recognize whats similar and what's not.

The sound material is a pretext – although a very important one – in which we recognize the proportions of the foundation of the structure of a composition. The ear listens to those proportions in a similar way as the geometric shapes show structures of repeated reasoning. The proportions offer the composition a particular topology – the sense of internal space, its organization, its density, the distance between objects in that space, its continuity and the fragmentation in regions. Definitively, proportions establish a metaphorical meaning.

Inversely, even the most seemingly remote examples of this process of building proportions can be categorized by using a metaphor. If not with a metaphor of beauty, it could be with the metaphor of the sublime, as it occurs when we contrast the examples of compositions of the classic period to the ones of the romantic period. While the classic beauty seems to calm the conflicts of the rational mind – it also reveals, illuminates, affirms and charms – the experience of the romantic beauty seems to have more to do with the opposite. The classic beauty appears with such form that the object that we admire adapts perfectly to our aesthetic judgement and what it produces is mutual affirmation between the observer and the contemplated object. Classic beauty may seem to have a place out there, and a public meaning. Romantic beauty, on the other hand, doesn't seem to construct an objective world any further than the personal or individual interest. On the contrary, the romantic perception of beauty invites to a private meditation. Its proportions do not offer great satisfaction for themselves. The romantic music provokes uneasiness, and sometimes it bothers. The reason is that it doesnt invite objective coherence but only individual exploration and by doing so, it submerges us, it defies us, it confuses us. This is what the experience of the sublime consists of, an experience which is primarily subjective, nonexisting in the exterior world with independence of the observer and the listener.

The construction of metaphors is key if we want to understand multiple connections, often hidden and historically silenced, in different fields of knowledge, like the ones we are discussing here. It is not a matter of strategic or heuristic procedure, we are not talking about "discoveries" nor "verifications" as the philosophers of science often do. Even though the metaphoric formulation has been defended in this field (as in the work of Mary Hesse), what is essential for us is that a formal procedure (building proportions) is naturally transformed into a phenomenological one (the experience of such proportions in two very different fields of knowledge) leading us to cognitive and emotional comprehension.

Its an eror to think of mathematics as the logical-deductive enterprise par excellence – a mistake so illogical as is to think of the etymology of the word "music" and believe that the activity to which the word refers to can only occur with the help of the muses. Both disciplines – music and mathematics – are closely linked, not by a formal perspective, but also – and this is what we wanted to show you in this paper – from a phenomenological standpoint.

2 Conclusion

We have briefly explored the phenomenological relationship between music and mathematics. We have given some examples that may clearly validate the contribution of math's intuitive dimension to some of the most important historical discoveries of this discipline. We have been able to demonstrate how these intuitive visions present an order that we might associate with the idea of beauty, as we refer to an aesthetic way of thinking. We have also shown that this way of reasoning, commonly attached to music and mathematics, is not a mere abstract construct, but it has practical consequences, since beauty contributes to truth and truth contributes to beauty. Finally, we have argued that what essentially unites both disciplines is the process of creative research and the constructions of metaphors, a process that permits the translation of symbolic codes. We believe this is one of the keys of the transdisciplinary process of learning and cognition, that is, how to build symbolic codes through metaphors that are common to diverse disciplinary contexts. In the case of the cognitive and phenomenological overlaps between music and mathematics, transdisciplinarity rises as the binding material allowing a common understanding of both knowledge areas.

References

[1] Eagleton, T. (1989). The Ideology of the Aesthetic. *The Rhetoric of Interpretation and the Interpretation of Rhetoric, 9* (2), 75.

[2] Hadamard, J. (1954). *An essay on the psychology of invention in the mathematical field.* Courier Corporation.

[3] Hanslick, E., Cohen, G., & Weitz, M. (1957). *The beautiful in music* (p. 29). New York: Liberal Arts Press.

[4] Kant, I. (1952) The Critique of Judgement (Trad. J.C. Meredith), Oxford: Oxford University Press.

[5] Kramer, L. (1990). Music as cultural practice, 1800-1900.

[6] Struik, D.J. (1987). A Concise History of Mathematics, 4th ed. New York: Dover.

Dr. Gerardo del Cerro Santamaría is Director of Assessment and Innovation and Professor of Social Science at The Cooper Union for the Advancement of Science and Art in Manhattan. He is also a Visiting Professor at MIT and Senior Research Fellow at the University of the Basque Country in Spain. He is widely recognized for his scholarly contributions to various fields of endeavor. Dr. del Cerro Santamara is the author of several books, book chapters, journal articles and encyclopedia articles about urbanism and globalization, urban megaprojects, architecture and the social sciences, innovation research, urban environmental engineering and program and organizational evaluation. Dr. Gerardo del Cerro Santamara has been awared the Doctoral Medal of the Universidad Autnoma de Madrid (Spain), the Alumni Association Award of the Colegio Mayor Ximnez de Cisneros, and has been named Honoraty Member of the Department of Electrical and Computing Engineering at The Cooper Union. He is a great music lover, holds a classical piano degree from the Royal Conservatory of Music in Madrid (Spain)and plays regularly in New York with fellow musicians and friends.