Understanding the Reliability of the Complex System having two Standby Units: A Transdisciplinary Approach

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Abstract: *The main objective of this study is to thoroughly investigate the reliability of a complex system that consists of four components and two standby units. The complicated system is modeled using the Markov technique. The study formulates differential equations and solves them using a matrix method with the assistance of a computer program. Statistical methods, such as correlation and regression, are applied using SPSS software to understand the relationship between time and the reliability of this complex system. A statistical model is utilized to determine the system's reliability, identify potentially faulty components, and prioritize the protection of the most critical elements. The presentation of significant findings involves conducting a comprehensive analysis using tables and graphs created with Python programming language. The outcomes of this study aim to assist in identifying crucial units in various process industries, thereby facilitating the establishment of effective management policies and strategies for the optimal operation of complex and large-scale systems in the industry. This paper incorporates numerous disciplines, viewpoints, and methodologies to examine and enhance reliability in process industries. Reliability can indeed benefit from a matrix approach, which can be considered transdisciplinary. This study serves as both an introductory guide to reliability analysis for novice researchers and a survey highlighting the latest advancements in the field for experienced researchers.*

Keywords: Matrix method, Differential equations, Statistical model, large-Scale systems, Transdisciplinary.

1 Introduction

The development and improvement of modern civilization heavily depend on the effectiveness of engineering systems. These systems span several areas, including industries, networks of infrastructure, and critical equipment like gas turbines. Their complex social presence has a significant impact on everyday lives as well as economic perspectives. Over time, engineering systems naturally deteriorate due to operational and environmental influences. Their total performance is reduced as a result of this steady deterioration, or in the worst case scenario, the system fails entirely. The uninterrupted operation of these systems—also known as reliability—is of utmost significance because of its close relationship to both safety and financial stability. During the system design and maintenance phases, prudent decisions need to be made in order to maintain this reliability. Critical failures in production systems can have disastrous effects on the environment, the economy, and the welfare of people. Consequently, it is necessary to take the removal of crucial failure modes into account, preferably at the pre-design stage. Compared to corrective actions during the manufacturing or testing phases, the proactive incorporation of reliability elements at this point is far more economical. It is essential that reliability considerations be incorporated early in the design phase of complex systems. This proactive strategy reduces the risks connected with system failures and proactively avoids possible failures. Therefore, giving reliability concerns top priority in the early stages of design is a fundamental component in the creation of resilient and long-lasting engineering systems. A transdisciplinary perspective recognizes the interdependence of numerous components within a complex system. It takes into account not only technical considerations, but also social, cultural, economic, environmental, and human factors that affect reliability. Experts from many areas such as engineering, computer science, sociology, psychology, economics, ecology, and others work together to provide insights into various aspects of the system. This cooperative effort improves understanding of the system's complexities and deficiencies.

In recent years, there has been a notable surge in the significance attributed to the reliability analysis of complex systems utilized within the process industry. Notably, reliability theory professionals have given mechanical systems a great deal of attention. In his academic paper, Michelsen [1] presented a strong argument in favor of the generalization and development of reliability technologies, particularly in the context of the process industry. Habchi [2] focused on enhancing the existing procedures related to this technique in order to improve the process for assessing the reliability of suspended testing approaches. Kiureghian and Ditlevson [3] examined the availability, dependability, and downtime of systems with repairable components in a different research. Their study highlighted the importance of these variables in comprehending system performance by shedding light on the complex dynamics present in systems with repairable components. An extensive investigation about the availability of an automobile system was carried out by Singh et al. [4]. Their research attempted to provide a thorough assessment of the system's availability, offering insightful information on the reliability of automotive systems. Amiri et al. [5] presented an innovative transient analysis approach that focuses on availability and survivability assessment in a system with identical components and a single repairman. This methodological suggestion presented novel techniques for evaluating the reliability of a system in temporary circumstances. Furthermore, using both available information and uncertain data, Komal et al. [6] descended into the estimation of the Reliability, Availability, and Maintainability (RAM) properties of industrial systems. Their method, which included Genetic Algorithms based Lambda-Tau (GABLT), sought to extract important RAM characteristics and offer a thorough framework for assessing system reliability. A model was presented by Filieri et al. [7] to describe how parts are disposed to produce, distribute, modify, or mask different failure modes. Many scholars have used different approaches to look at the reliability of complex and large-scale systems. A two-unit system model for assessing and monitoring a continuous casting plant's reliability was presented by Mathew et al. [8]. They explored the process of obtaining crucial reliability indices by skillfully applying regeneration point techniques and semi-Markov processes, which helped to construct a robust model. A technique based on a graph-theoretic system approach was proposed by Upadhyay et al. [9] to address the reliability study of Component Based Software Systems (CBSS). Their work was centered on reliability, which they described as a mechanical system's capacity to function flawlessly over extended periods of time without requiring frequent repairs, modifications, or component replacements. Calixto [10] validated the notion of system performance indices that include production efficiency, availability, and reliability and explained how to predict these performance indicators. A Markov chain-based error propagation model was presented by Tian et al. [11] to assess component-based software systems' reliability and protect their vital components. Ren and Guo [12] described a technique for computing dependability that incorporates an error propagation model based on Markov chains. Moreover, Taj and Rizwan [13] conducted a thorough evaluation of the literature on the modeling and analysis of complex industrial systems reliability. They evaluated the many kinds of systems that were examined, the range of assumptions and operational circumstances that were taken into account, the research methods that were used, and the final results of these investigations. Accelerated Life Testing (ALT) was used by Moustafa et al. [14] to present a novel methodology for assessing the reliability of systems with many components. In a similar vein to assess the effectiveness of a water circulation system in a coal-fired power station, Jagtap et al. [15] carried out a thorough RAM (Reliability, Availability, and Maintainability) analysis. In the meanwhile, Maihulla et al. [16] examined a three-system reverse osmosis filtering system's reliability metrics. Wang et al. [17] clarified the investigation of modern electromechanical systems and a thorough review of current developments in reliability theories, models, procedures, and related software tools. They carefully examined the benefits and drawbacks of different approaches in their study. The literature on reliability was the primary topic of Farahani et al.'s [18] study, with a specific emphasis on Markov and semi-Markov models. Taj and Rizwan [19] focused on reliability evaluations while modeling and evaluating a complicated industrial system with two continuously operating units. Balushi [20] carried out reliability evaluations of power transformers in the context of power distribution networks, using sensitivity analysis to determine the impact of different variables on reliability indicators. Zaidi [21] examined many numerical techniques and mathematical tools intended to provide a more reliable framework for evaluating the transient reliability of systems. Zaidi investigated the solutions to differential equations regulating a transient state in a three-component system using MATLAB 7.8.0 (R2009a) and three alternative approaches: the Laplace Transform technique, the Matrix approach, and simple integration.

This study employs the solution of differential equations controlling the behavior of the system to evaluate the reliability of a complicated system with four components and two backup units. A statistical model that shows the probability associated with different system states over time is built using a Markovian technique. The two key variables in this model's structure are system state and time, which allow for a thorough representation of the dynamic behavior of the system. The conclusions drawn from this study have important ramifications for the assessment of reliability in large and complex systems. The reliability of the system is clearly and thoroughly understood by thorough statistical analysis. Statistical approaches are used to help provide a thorough picture of how the system behaves and performs in various operating settings.

The study's results provide insightful information that may be used to evaluate and examine large-scale systems. This technique gives insights that may be applied to assess and improve the reliability of several different complex systems in addition to providing a methodological foundation for understanding the reliability of complex systems. When seen through a transdisciplinary lens, reliability in complex systems entails a complete technique that integrates information, experience and approaches from numerous disciplines to comprehend and improve the reliability of complicated systems.

The paper is structured into seven distinct sections, each serving a crucial role in presenting and analyzing the complexities of the subject matter. The introductory paragraph lays the foundation for the study, providing an overview and context for the subsequent sections. Section 2 of the paper delves into the assumptions and notations pertinent to the complex system under investigation. This section serves to define the parameters, assumptions, and symbols employed throughout the study, establishing a common understanding and framework for further exploration. Moving to Section 3, the formulation of the differential equations governing the behavior of the complex system is presented using mnemonic rules.

This section is crucial as it elucidates the mathematical framework employed to model the system's dynamics, providing a clear understanding of its operational principles. Section 4 of the article focuses on the analysis of reliability in a transient state. Here, the transient behavior of the system is explored and analyzed, shedding light on how the system performs and evolves during transient phases, which are critical in understanding its overall reliability. In Section 5, the application of statistical methods such as correlation and regression techniques is employed. This section aims to harness statistical tools to further comprehend the interrelationships between different system parameters, offering insights into their influences on the system's reliability. The interpretation of the obtained results is articulated. This section critically evaluates and explains the significance of the findings derived from the analysis conducted in previous sections. It provides a comprehensive interpretation, drawing connections between the theoretical framework and practical implications. Section 6 encapsulates the Conclusions. Here, the key findings and implications are summarized, providing a synthesis of the study's outcomes. Finally, Section 7 depicts the discussion of how multidisciplinary approach expresses itself in tackling reliability in complex systems.

2 System Description

2.1 Assumptions and Notations

Let P, Q, R, and S represent the four components of the complex system, where R and S have standby units for support. The following assumptions are considered:

- All components are initially in the operating condition and are in a good state.
- Each component has two states: good and failed, except for R and S that have three states: good, reduced, and failed.
- Every component returns to a good-as-new state after repair.
- The failure rates and repair rates for all components remain constant.
- Failure and repair events for each component are statistically independent.
- Immediate repair of a failed component commences as soon as it fails.

The following notations are used in the present analysis of the complex system:

P, Q, R, and S represent good working states of complex system.

p, q, r, and s represent failed states of a complex system.

R_1 *and* S_1 represent reduced states of complex system.

λ_i ($1\leq i \leq 6$) depict the failure rates of states P, Q, R, R_1, S and S_1.

μ_i ($1\leq i \leq 6$) depict the repair rates of states P, Q, R, R_1, S and S_1.

$P_i(t)$ are the state probabilities that the system is in i^th^ state at time t.

2.2 Transition Diagram

Figure 1 shows the system's transition diagram based on the notations and assumptions mentioned above. To create the differential equations governing this complex system, the mnemonic rule is used, employing this figure as its basis.

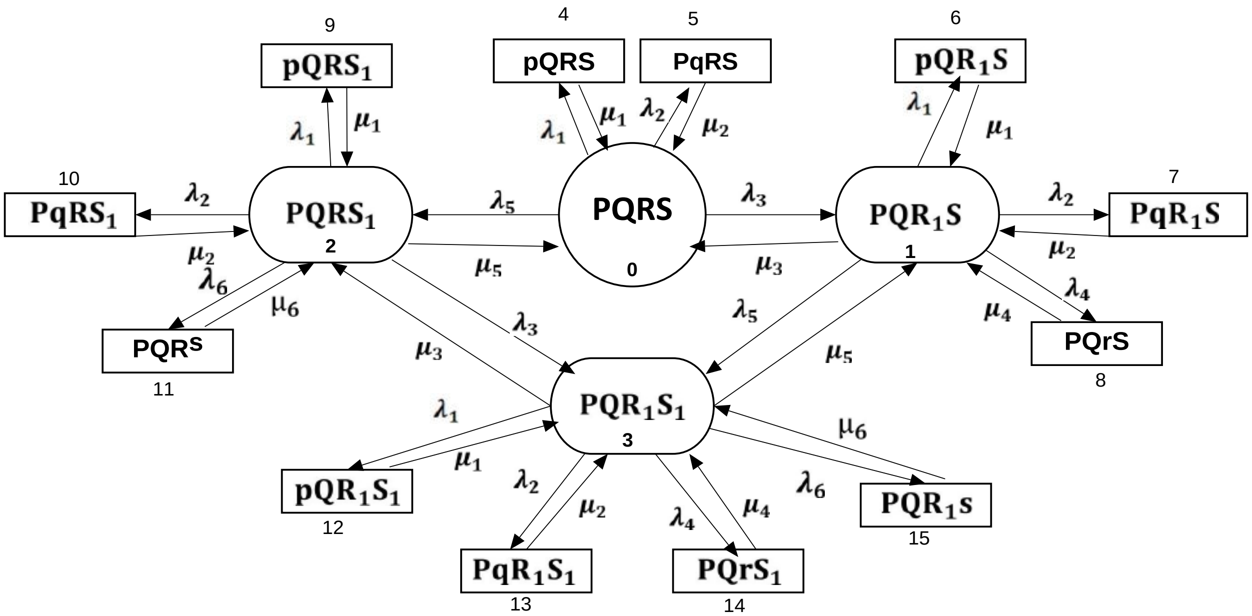

FIGURE: 1 TRANSITION DIAGRAM

3 Mathematical Modeling

To determine the reliability of a complex system in its transient state, the mathematical formulation is carried out using the mnemonic rule for four components with two standby units. The mnemonic rule often refers to a technique of assessing a complex system's reliability by taking into account all possible states for the system. This approach entails listing all conceivable states of the system and computing its reliability in each, as in the instance of a four-component system with two standby units. In order to assess the total reliability of the complex system in its transient state—which takes into consideration standby units and different component failure scenarios—this approach entails thoroughly identifying and evaluating every conceivable combination of component states. The system's reliability is determined by the use of equations including component failure rates, repair rates, and probability of various states.

3.1 Transient State

The differential-difference equations obtained from the state transition diagram using Mnemonic rule at time $(t + \Delta t)$ are:

$$P_0(t+\Delta t) = [1 - \lambda_1 \Delta t - \lambda_2 \Delta t - \lambda_3 \Delta t - \lambda_5 \Delta t]P_0(t) + P_4(t)\mu_1 \Delta t + P_5(t)\mu_2 \Delta t + P_1(t)\mu_3 \Delta t + P_2(t)\mu_5 \Delta t$$

$$P_0(t+\Delta t) - P_0(t) = [-\lambda_1 \Delta t - \lambda_2 \Delta t - \lambda_3 \Delta t - \lambda_5 \Delta t]P_0(t) + P_4(t)\mu_1 \Delta t + P_5(t)\mu_2 \Delta t + P_1(t)\mu_3 \Delta t + P_2(t)\mu_5 \Delta t$$

Dividing both sides by Δt, we get:

$$\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = [-\lambda_1 - \lambda_2 - \lambda_3 - \lambda_5]P_0(t) + P_4(t)\mu_1 + P_5(t)\mu_2 + P_1(t)\mu_3 + P_2(t)\mu_5$$

Taking $\Delta t \to 0$,

$$P_0'(t) = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_5)P_0(t) + \mu_1 P_4(t) + \mu_2 P_5(t) + \mu_3 P_1(t) + \mu_5 P_2(t)$$

$$P_0'(t) + (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_5)P_0(t) = \mu_1 P_4(t) + \mu_2 P_5(t) + \mu_3 P_1(t) + \mu_5 P_2(t)$$

$$P_0'(t) + B_1 P_0(t) = \mu_1 P_4(t) + \mu_2 P_5(t) + \mu_3 P_1(t) + \mu_5 P_2(t) \quad (1)$$

Similarly:

$$P_1'(t) + (\lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \mu_3)P_1(t) = \mu_1 P_6(t) + \mu_2 P_7(t) + \mu_4 P_8(t) + \mu_5 P_3(t) + \lambda_3 P_0(t)$$

$$P_1'(t) + B_2 P_1(t) = \mu_1 P_6(t) + \mu_2 P_7(t) + \mu_4 P_8(t) + \mu_5 P_3(t) + \lambda_3 P_0(t) \quad (2)$$

$$P_2'(t) + (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \mu_5)P_2(t) = \mu_1 P_9(t) + \mu_2 P_{10}(t) + \mu_3 P_3(t) + \mu_6 P_{11}(t) + \lambda_5 P_0(t)$$

$$P_2'(t) + B_3 P_2(t) = \mu_1 P_9(t) + \mu_2 P_{10}(t) + \mu_3 P_3(t) + \mu_6 P_{11}(t) + \lambda_5 P_0(t) \quad (3)$$

$$P_3'(t) + (\lambda_1 + \lambda_2 + \lambda_4 + \lambda_6 + \mu_3 + \mu_5)P_3(t) = \mu_1 P_{12}(t) + \mu_2 P_{13}(t) + \mu_4 P_{14}(t) + \mu_6 P_{15}(t) + \lambda_3 P_2(t) + \lambda_5 P_1(t)$$

$$P_3'(t) + B_4 P_3(t) = \mu_1 P_{12}(t) + \mu_2 P_{13}(t) + \mu_4 P_{14}(t) + \mu_6 P_{15}(t) + \lambda_3 P_2(t) + \lambda_5 P_1(t) \quad (4)$$

$$P_{3+i}'(t) + \mu_i P_{3+i}(t) = \lambda_i P_0(t), i = 1, 2 \quad (5)$$

$$P_{5+i}'(t) + \mu_i P_{5+i}(t) = \lambda_i P_1(t), i = 1, 2 \quad (6)$$

$$P_8'(t) + \mu_4 P_8(t) = \lambda_4 P_1(t) \quad (7)$$

$$P_{8+i}'(t) + \mu_i P_{8+i}(t) = \lambda_i P_2(t), i = 1, 2 \quad (8)$$

$$P_{11}'(t) + \mu_6 P_{11}(t) = \lambda_6 P_2(t) \quad (9)$$

$$P_{11+i}'(t) + \mu_i P_{11+i}(t) = \lambda_i P_3(t), i = 1, 2 \quad (10)$$

$$P_{14}'(t) + \mu_4 P_{14}(t) = \lambda_4 P_3(t) \quad (11)$$

$$P'_{15}(t) + \mu_6 P_{15}(t) = \lambda_6 P_3(t) \tag{12}$$

Where:

$B_1 = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_5)$

$B_2 = (\lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \mu_3)$

$B_3 = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \mu_5)$

$B_4 = (\lambda_1 + \lambda_2 + \lambda_4 + \lambda_6 + \mu_3 + \mu_5)$

With initial conditions at time t = 0

$$P_i(t) = 1, for\ i = 0$$

$$P_i(t) = 0\ for\ i \neq 0$$

4 Reliability Analysis in Transient State

There are several analytical techniques that may be used to solve governing differential equations and determine the reliability of a system, including as Laplace transforms, Lagrange's approach, and Runge-Kutta methods. Although these methods are simple, they might not be advised for complex systems with a high number of governing differential equations. It is crucial to recognize that a great deal of work has gone into investigating different approaches to solving these kinds of differential equation systems. My motivation originates from the studies carried out in this field, which made me choose to use a matrix-based approach, made possible by software, to solve these differential equations. This method provides a methodical and computationally feasible means of solving differential equations by utilizing the ability of matrix computations to handle complicated systems effectively. By converting the differential equation system into a matrix form, the matrix approach makes it possible to solve the equations numerically through the use of computing tools. Using specialized software for differential equation solutions and matrix operations, this method may solve complex problems that are difficult or impossible to solve using standard analytical methods. The matrix-based method has benefits in terms of handling complex interactions between system components, scalability to more expansive systems, and computing efficiency. The use of this approach seeks to improve the accuracy and usefulness of reliability evaluations in many engineering and scientific applications by offering a more reliable and efficient way to solve the differential equations controlling the reliability of complex systems.

4.1 Matrix Method

When the coefficient matrix of a system of linear differential equations is constant, this technique is especially helpful. In terms of eigenvalues and eigenvectors, it offers a generic solution that facilitates the handling and analysis of complicated systems. Remember that this technique works best with matrices that are diagonalizable, or that is, matrices that include all of their linearly independent eigenvectors.

Time Dependent Case: The system of differential difference equations may be written as

$$(\theta I - B)\bar{P}(u,t) = \bar{0}$$

where $\theta = \frac{d}{dt}$, $\bar{0}$ is the null matrix, matrix B is the matrix of the coefficients of the probability states $P(u,t)$ and I or I_n is the identity matrix of order n.

Matrix B =

$$\begin{bmatrix}
-B_1 & \mu_3 & \mu_5 & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & -B_2 & 0 & 0 & 0 & 0 & \mu_1 & \mu_2 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_5 & 0 & -B_3 & \mu_3 & 0 & 0 & 0 & 0 & 0 & \mu_1 & \mu_2 & \mu_6 & 0 & 0 & 0 & 0 \\
0 & \lambda_5 & \lambda_3 & -B_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & \mu_2 & \mu_4 & \mu_6 \\
\lambda_1 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_2 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_1 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_4 & 0 \\
0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_6
\end{bmatrix}$$

Let $\mathrm{P(u,t)}$ denote the probability of the system at time t in the state u. If the number of all the possible transition states of a complex system are 'n' than the system of differential difference equations may be written as:

$$(\theta I - B)\bar{P}(u,t) = \bar{0}$$

where $\theta = \frac{d}{dt}, \theta$ used for differentiation, $\bar{0}$ is the null matrix, matrix B is the matrix of the coefficients of the probability states $P(u,t), u = 1, 2, 3, \dots \dots \dots n$

$\bar{P}(u,t) = [P(1,t)P(2,t) \dots \dots \dots . . P(n,t)]^T$ and I_n is the identity matrix of order n.

Let G be the matrix such that $G^{-1}BG = D$ where $D = (d_1, d_2 \dots \dots \dots d_n)$ be the diagonal matrix of eigen values of the matrix B.

$$P(1,t) = 1 + a_{11}t + b_{11}\frac{t^2}{2!} + \cdots$$

$$P(2,t) = a_{21}t + b_{21}\frac{t^2}{2!} + \cdots$$

……………………………

……………………………

$$P(n,t) = a_{n1}t + b_{n1}\frac{t^2}{2!} + \cdots$$

If $P(1,t), P(2,t), \dots \dots \dots \dots P(i,t)$ are the working states of a system, then Reliability is

$$\mathrm{R}(t) = P(1,t) + P(2,t) + \cdots \dots \dots \dots . . + P(i,t)$$

$$= 1 + (a_{11} + a_{21} + \cdots \dots \dots . + a_{i1}) + (b_{11} + b_{21} + \cdots \dots \dots . + b_{i1})\frac{t^2}{2!} + \cdots \dots$$

For various combinations of the repair and failure rates of the units, numerical computations have been done from time $t = 0$ to $t = 100$ days. The system's reliability $R(t)$ can be calculated by:

$$R(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t)$$

Taking the values of different parameters as $\lambda_1 = 0.002, \lambda_2 = 0.002$, $\lambda_3 = 0.004$, $\lambda_4 = 0.002$, $\lambda_5 = 0.004$, $\lambda_6 = 0.002$, $\mu_1 = 0.02$, $\mu_2 = 0.015$, $\mu_3 = 0.01$, $\mu_4 = 0.03$, $\mu_5 = 0.01$ and $\mu_6 = 0.025$. The following table is developed using above different values of failure and repair rates.

Table 1: Variation of Reliability with Time

Time	10	20	30	40	50
Reliability	0.963337	0.932719	0.907256	0.886152	0.868722
Time	60	70	80	90	100
Reliability	0.854448	0.843166	0.835570	0.834395	0.833211

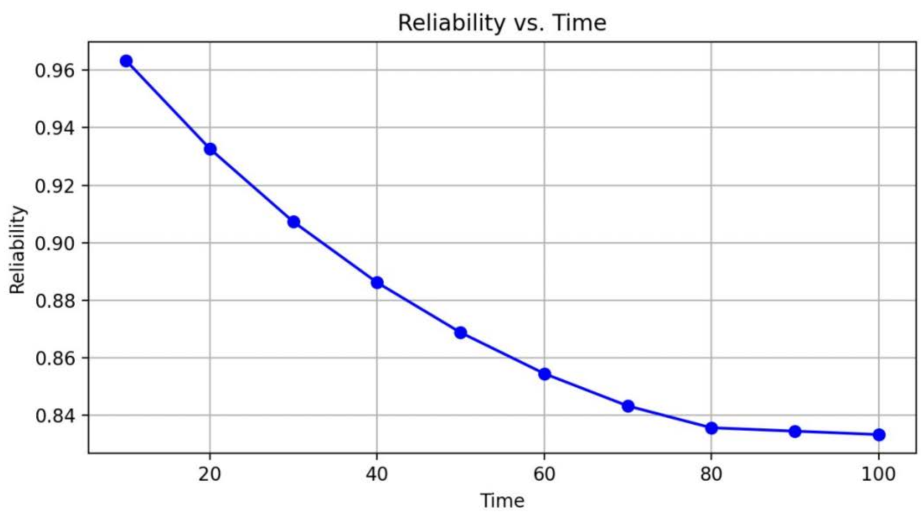

FIGURE 2: Variation of Reliability with time

5 Statistical Techniques

5.1 Correlation Analysis

In the context of a complex system, the study applied SPSS software to investigate the relationship between two important variables: time and reliability. A correlation coefficient (r) value of -0.95 supports the strong negative association between time and reliability that was found after a thorough study. Time and reliability have a very strong negative linear relationship, demonstrated by the correlation value of -0.95. Such a coefficient suggests that there is a significant propensity for the system's reliability to decline with time. Furthermore, the statistical significance of this relationship was rigorously established through hypothesis testing. The calculated p-value, being less than 0.001, attests to an exceedingly high level of significance. This indicates that the observed correlation between time and reliability is highly improbable to have arisen due to random chance alone. Instead, it strongly suggests a genuine and significant negative relationship between these variables.

Table 2: Correlation Analysis

		Time	Reliability
Time	Pearson Correlation	1	-.950**
	Sig. (2-tailed)		<.001
	N	10	10
Reliability	Pearson Correlation	-.950**	1
	Sig. (2-tailed)	<.001	
	N	10	10

**. Correlation is significant at the 0.01 level (2-tailed).

5.2 Regression Analysis

Using SPSS software, the complex system's regression analysis between time and reliability is conducted.

Table 3: Regression Results

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.950[a]	.903	.890	.01510

a. Predictors: (Constant), Time

The outcomes depicted in Table 3, specifically the regression analysis, underscore a substantial relationship between time and reliability within the complex system. The model showcases a noteworthy R-square value of 0.903, signifying that approximately 90.3% of the variability observed in reliability can be elucidated by changes over time. This high R-square value implies a robust association between these two variables, suggesting that as time progresses, there is a notable impact on the system's reliability, as inferred from the model.

Table 4: ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.017	1	.017	74.096	<.001[b]
	Residual	.002	8	.000		
	Total	.019	9			

a. Dependent Variable: Reliability

b. Predictors: (Constant), Time

Table 4, which presents the ANOVA results, further fortifies this observation. The analysis illustrates a statistically significant relationship between time and reliability, as indicated by the p-value of less than 0.01 (denoted as <.001). This finding supports the inference drawn from the regression analysis, confirming that the changes in reliability are indeed linked to the passage of time within the context of the studied complex system.

Table 5: Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	.955	.010		92.547	<.001
	Time	-.001	.000	-.950	-8.608	<.001

a. Dependent Variable: Reliability

The results of the coefficients are shown in table 5. As indicated that beta value is -0.95, which means that the change in time by one unit will bring 0.95 units change in the reliability. Additionally, the beta value is negative, indicating a negative relationship between reliability and time. In other words, when time increases by one unit the reliability will decrease by 0.95 units.

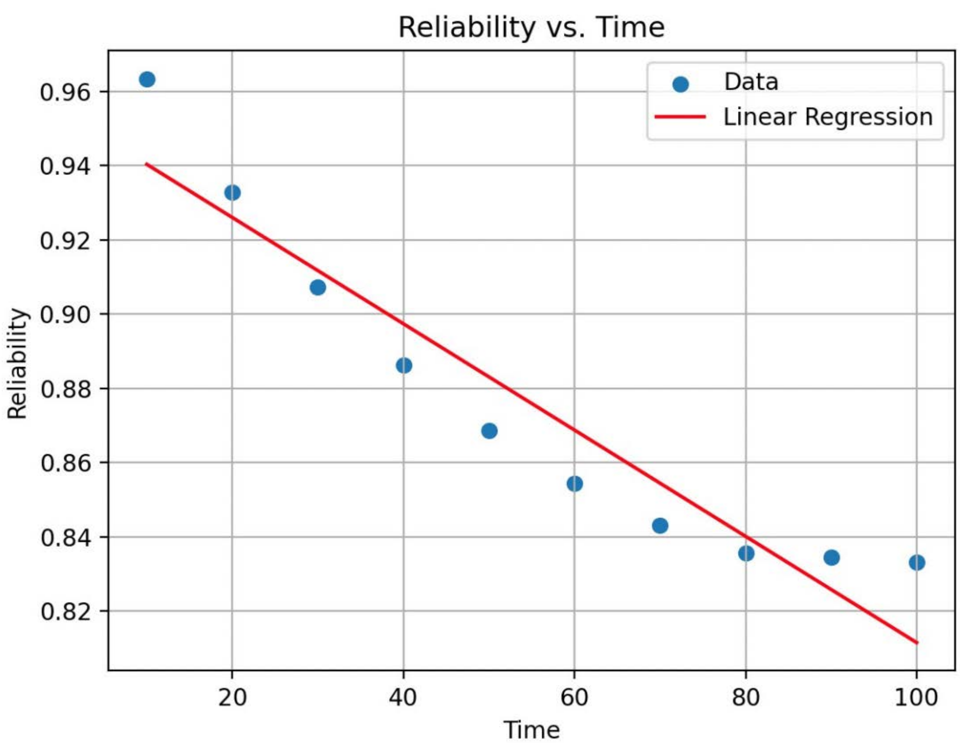

FIGURE 3: Estimated Regression line

Estimated Regression Model: $\text{Reliability} = 0.955 - 0.001(\text{Time})$

5.3 Interpretation of the Results

In the present research, a four component complex system's strength is evaluated using correlation and regression analysis. The statistical results are obtained using the SPSS program. The -0.95 correlation coefficient shows a strong negative correlation between time and reliability. In order to illustrate how reliability qualities change over time, a tabular depiction of the statistical findings is also provided. Table 1 and Figure 2 both display the variance in reliability with reference to time. Reliability begins to steadily decrease after a significant amount of time and eventually becomes nearly steady. The research design needs to take into account the possible impact of external influences or confounding variables that might affect the complex system's time and reliability. In summary, the thorough analysis carried out with SPSS software has shown a strong and statistically significant negative relationship between time and reliability in the complex system under study, meaning that as time progresses, the system's reliability significantly declines. Collectively, the findings of the ANOVA (Table 4) and the regression analysis (Table 3) agree to show that time and reliability in the complex system under study have a strong and statistically significant relationship. The strong influence of time on the system's dependability is confirmed by the high R-square value and the considerably low p-value, which emphasizes the significance of temporal considerations in comprehending and perhaps regulating reliability within this system.

6 Conclusions

In this study, a unique approach to reliability modeling is presented, which is intended to examine system performance, durability, and effectiveness. This study presents a detailed description of a software tool for evaluating the reliability of complex systems. This software makes large-scale system computing easier, which makes reliability estimations easier. Understanding the critical role reliability analysis plays, it becomes necessary for every process industry aiming for maximum production efficiency to integrate reliability analysis into operations. Making use of the knowledge in this article has the potential to improve operational processes' quality and productivity. The presented approach, in its current form, demonstrates the ability to handle intricate systems including a significant number of differential equations. The study explores the link between time and reliability using regression and correlation analysis. By identifying key components, this method has the potential to greatly enhance the performance of complex systems and facilitate the execution of reform plans.

The study's findings highlight the value of reliability modeling in examining complex systems' resilience and potential for improved performance. Given a collection of parametric variables, the statistical studies shown in Tables 2, 3, 4, and 5 validate the capacity to predict complex system behavior with accuracy across time. Decision-makers can better understand complex systems by using the suggested reliability analysis, which clarifies their complexities. In conclusion, the clearly defined reliability modeling technique is a useful instrument for decision-makers as it provides understanding of intricate systems and an organized approach to schedule and maximize maintenance tasks for increased operational efficiency.

7 Discussion

The investigation of a complex system's reliability has been done using a transdisciplinary approach that integrates many disciplines, techniques, and points of view. This study exhibits the following transdisciplinary characteristics:

- Multidisciplinary Modeling Techniques: Mathematical and probability theory concepts are applied while modeling the complex system using the Markov approach. Markov models are frequently used to illustrate the integration of different disciplines in a variety of fields, such as statistics, computer science, and engineering.

- Statistical Analysis using SPSS: Data analysis and statistical knowledge are required for the use of statistical techniques such as regression and correlation as well as for using SPSS software. Incorporating techniques from data science and statistics gives the study an additional statistical dimension.
- Differential Equations and Matrix Methods: Mathematical and computational skills are needed to formulate and solve differential equations using matrix methods. Computer programming, numerical analysis, and mathematical modeling are all included in this multidisciplinary approach.
- Utilization of Python Programming: Using the Python programming language to create graphs gives the research a computational element. Python is frequently used to combine computer science with data analytics in the display and analysis of data across a wide range of fields.
- Determining Potentially Faulty Components: This requires not just a technical understanding of the components of the system but also expertise in failure analysis, which may be gleaned from the disciplines of reliability engineering, material science, and engineering.
- Application in Process Industries: The goal of the study is to support the identification of key compo nents in a range of process industries. This necessitates combining perspectives from the fields of industrial engineering and management with a grasp of systems analysis, operational management, and industrial processes.
- Educational and Survey elements: The research addresses educational and survey elements by acting as a guide for new researchers and showcasing the most recent developments for experienced researchers. This encompasses the sharing of knowledge among experts at various levels, encouraging learning and awareness across a variety of disciplines.
- Implications for Policy and Management: By aiming to aid in establishing effective management policies and strategies, this research goes beyond technical details by including perspectives from the domains of policy-making, industrial management, and organizational behavior.

Finally, the transdisciplinary nature of this research is demonstrated by the combination of different approaches and insights from different fields, including computer science, engineering, mathematics, statistics, and management, with the ultimate goal of improving the reliability of complex systems in process industries.

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